7.3

Our Biggest Sale of the Season!

Systems with More Than Two Linear Inequalities

LEARNING GOALS

In this lesson, you will:

- Solve systems of linear inequalities.
- Maximize linear expressions on a region in the coordinate plane.

You've probably heard the sales pitches on the radio or seen the advertisements on television—you've probably even seen them on-line. Yes, it's the biggest sale of the season! Black Friday is here! This Friday in November has been the busiest shopping day of the year for at least a decade and that trend does not seem to be changing any time soon. Black Friday is named as such because this tends to be the day on which retailers begin to turn a profit for the year, which is known as "being in the black." But how can a day known for huge blowout sales be a day when retailers begin making a profit? Due to the extended store hours (some stores open as early as midnight!) and the great deals, the stores tend to see more sales that usual. So even though they have cut their prices, so much more is sold that it makes up for the reduced prices.

Do you think Black Friday deals are worth the crowds and stress that come along with the sales? How can you determine if a sale is as good as it sounds?

PROBLEM 1   **More Than Two? Is That Even Possible?**

Miguel's eye doctor informed him that he needs glasses. However, Miguel shouldn't fret as *I've Got My Eye-Glasses On You* is having a sale on all eyeglass frames. The advertisement in the window is as shown:

> **Save 60% to 75% On All Frames**
> **Regularly Priced at**
> **$120 − $360**

Previously, you solved a system containing two linear inequalities. However, systems can consist of more than two linear inequalities.

1. Use the advertisement to write two inequalities that represent the regular price of eyeglass frames. Let *r* represent the regular price of the frames.

2. Use the same advertisement to write two inequalities that represent the reduced price of the eyeglass frames. Let *s* represent the sales price of the frames in terms of *r*.

> When an item is 20% off the regular price, you can think of that item costing 80% of the regular price!

3. Heather wrote this system of linear inequalities for the problem situation.

> 👎 **Heather**
>
> $r \geq 120$
> $r \leq 360$
> $s \leq 0.6r$
> $s \geq 0.75r$

Explain why Heather's system of linear inequalities in incorrect.

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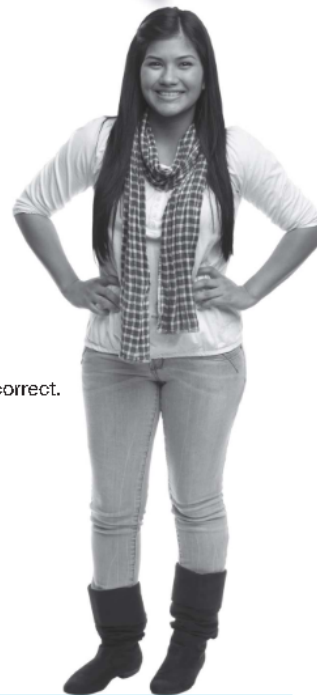
4. Graph each inequality on the grid shown. Represent the regular price of the glasses as the *x*-axis and the sale price as the *y*-axis. Use the information in the table shown to set the bounds and intervals for your graph. Do not forget to add a unit of measure for each axis label.

Variable Quantity	Lower Bound	Upper Bound	Interval
Regular Price	0	400	40
Sale Price	0	200	20
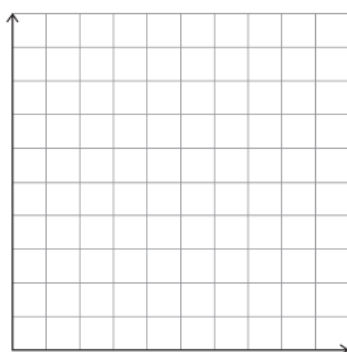
When graphing a system of linear inequalities, you must determine the portion of the graph that satisfies *all* the inequalities in the system.

5. Shade the portion of the graph that satisfies the system of linear inequalities. What shape does the solution region resemble?

6. Using your graph, about how much would Miguel expect to spend if he purchases eyeglasses that are regularly priced at $320?

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7. Miguel is definitely going to purchase a pair of eyeglasses that are on sale. What is the least amount of money Miguel can expect to spend? What is the greatest amount he can expect to spend?

8. Miguel decides on a pair of eyeglasses that are regularly priced at $240.

   a. Can Miguel expect to save more or less than $140 off the purchase price of this pair of eyeglasses? Use your graph to determine an approximate answer.

   The graph shows me the sale price of the eyeglasses, but how can I determine how much he will save?

   b. Use algebra to determine the greatest amount of money Miguel can save by purchasing eyeglasses that are regularly priced at $240.

PROBLEM 2   Give It Up For Macho Nacho

Maize Tortillas and Chips is introducing a new chip to their product line. The Macho Nacho claims to hold six times more cheese than any other tortilla chip on the market. However, the release of this monumental chip couldn't have come at a worse time. The failure of the Brussels sprout chip in 2011 has caused upper management to cut advertising and marketing budgets for the upcoming year. The vice president of Maize has announced that *all* advertising budgets less than $15,000 will be cut by a range of 10% to 32% in 2012.

1. Write a system of inequalities to represent this problem situation. Let *a* represent the advertising budget for 2011, and let *c* represent the advertising budget for 2012.

2. Graph each inequality on the grid. Represent the advertising budget for 2011 on the *x*-axis and the advertising budget for 2012 on the *y*-axis. Do not forget add labels and units of measure for each axis.

3. Use algebra to determine the points of intersection for the solution region on the coordinate plane. Then label these points on your graph.

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4. Predict the point of the solution region that will have the least value. Explain why this point has the least value.

5. Terrance claims that the greatest value in the solution region that satisfies the system is (15,000, 13,500). Shira thinks that Terrance is incorrect and claims that this point does *not* satisfy the system. Who is correct? Explain to the student who is incorrect how to correct his or her claim.

6. Jorge makes a claim about the shape of the solution region.

> **Jorge**
>
> The solution region of the system is a triangle.
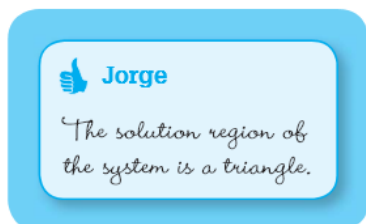
How can you prove that Jorge is correct?

PROBLEM 3   See! Systems With More Than
Two Inequalities *Aren't* Scary!

As you saw in Problems 1 and 2, inequalities can be helpful when determining a solution set
for a particular system of linear inequalities. The steps you should remember when solving a
system of linear inequalities with more than two inequalities are:

- Graph each linear inequality.

- Determine and shade the region that contains the solution sets that satisfy all the
  inequalities in the system.

- Determine all the points of intersection for the boundary lines that make the vertices of the
  solution region.

- Remember to use a closed point if the point is part of the solution region, and use an
  open dot if the point is not part of the solution region, but is a point of intersection.

1. Graph the solution set for each system of linear inequalities. Label all points of
   intersection of the boundary lines. Then determine a point the satisfies all the linear
   inequalities in the system.

a. $\begin{cases} x \geq 0 \\ y \geq 0 \\ x + y \leq 6 \\ y \geq 2x - 5 \end{cases}$
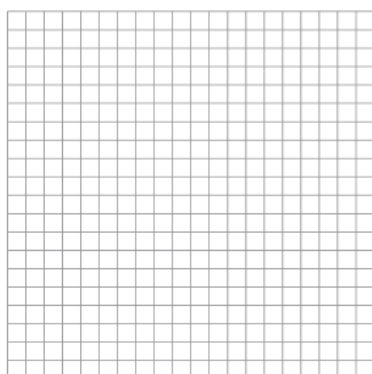
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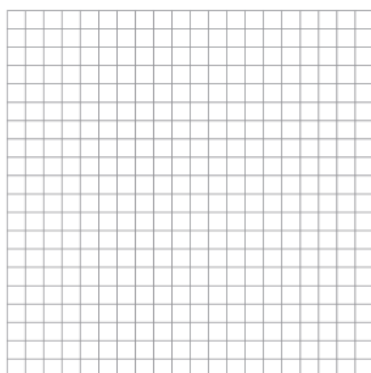
b. $\begin{cases} x \geq 0 \\ y \geq 0 \\ y \leq -2x + 8 \end{cases}$

c. $\begin{cases} y < 2x + 5 \\ x + y < 3 \\ 2x - 3y \leq 6 \end{cases}$

Is the point (2, 1) a solution of the system? Why or why not? Use algebra to support your answer.

Be prepared to share your solutions and methods.